
ANALYSES OF MATHEMATICAL PROBLEM SOLVING ABILITY OF PUBLIC JUNIOR HIGH SCHOOL STUDENTS

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Abstract

This article is written based on data from the prior research that support main research about The Effect of Problem-based Learning towards mathematical understanding ability (MUA) and mathematical problem solving ability (MPSA) of Public Junior High School (PJHS) students in Bandung. The prior research is intended to analyze the achievement of the students in solving mathematical problems and to collect information about the cause of various mistakes made by them in solving such problems. Mathematical problems is designed in MUA and MPSA tests. The kind of the test is essay test. The research results showed that some mistakes students made in solving mathematical problems are due to: (1) students' fluency in algebraic operation, (2) lack of mathematical prior knowledge, and (3) limited mathematical understanding and representation. Related to the students' achievement: the students' average score for MUA test is 39,95% and 32,46% for MPS test. So, it is suggested that the teachers give more attention to improve students' capability in solving algebraic problems, remind and reinforce students' mathematical prior knowledge periodically, and develop students' mathematical understanding & representation ability. In this article, students' performance in solving the tests described descriptively.

Keywords: Problem-based Learning, Mathematical Problem Solving, Mathematical Understanding.

A. BACKGROUND

Learning with understanding is a necessity. This is because the purpose of learning anything is to understand. Especially for mathematics as a hierarchical science understanding becomes increasingly important. This statement is in line with the objective of learning mathematics for the students at elementary school until senior high school (Depdiknas, 20016). Therefore, it becomes very important that schools first emphasize the learning of mathematics as an effort to gain an understanding of mathematical object. Hiebert & Carpenter (1992) confirm that since now, widely accepted idea in the mathematics education community is the students should grasp understanding in everything they learned. According to Hiebert & Carpenter (1992), a lot of mathematics education research intended to promote learning with understanding even though it looks very difficult to achieve.

Understanding occupies a strategic position because it underlies all mathematical process. However, understanding is supported by and closely interwoven with others mathematics standard process. For example, the representation of a concept or problem

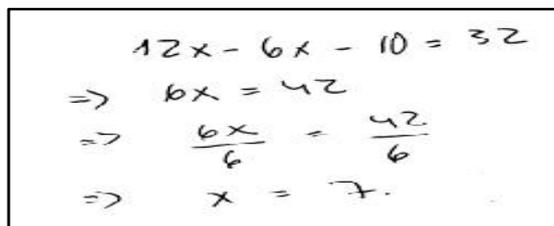
from different point of view will help one's understanding towards that concept or problem. The ability to associate ideas in mathematics also contributed to the achievement of deep understanding of the idea (Hiebert & Carpenter, 1992).

The results of the evaluation conducted by TIMSS (The Trends of Mathematical and Science Study) can be used as a reference of mathematics learning achievements (especially the achievement in mathematical understanding ability) of junior high school student in Indonesia. Through out the history of TIMSS, Indonesia has included students in grade 8 consecutively in 1999, 2003 and 2007 (Mullis, 2008). In all three times participations, Indonesian students' achievement is just around the category of low (400), which means the students have only partial knowledge of basic mathematics. An example of TIMSS' problem is presented below.

Problem 1. Find the x value of $12x - 10 = 6x + 32$

The international average achievement for this problem was 44%, while Indonesia is only 18% (Mullis, et al., 2000:76), significantly far below the international average. The task contained in problem 1 is complex enough. It requires skills in doing additive inverse so that the term that includes variables in that equation collected in one segment and the constant term in other segments. That is why the achievements of students to this problem is low.

This problem almost impossible to solve informally and even if it is done, it will be more difficult than solving it through algebraic manipulation. An other possible way can be tried by the students via try and error, that is by trying out numbers that may satisfy the equation, but it is likely to spend a long time. Meanwhile, the procedure pursued by students who already sits in PJHS grade 8 is a routine step as below.



$$\begin{aligned}
 12x - 6x - 10 &= 32 \\
 \Rightarrow 6x &= 42 \\
 \Rightarrow \frac{6x}{6} &= \frac{42}{6} \\
 \Rightarrow x &= 7
 \end{aligned}$$

Figure 1. Students' answer of Problem 1

Problem 2. Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils? Show your work.

Problem number 2 requires the students' competency in creating mathematical model, that is, represent mathematical problem into linear equations and solve it. The achievement of students from many countries on this matter is fairly low, it is only 8% for international average. This problem can be solved properly as much as 8% of Indonesian students.

Indonesian students' achievement for problem 1 and 2 indicated the students' lack of understanding and problem solving skills. To support TIMSS report, a set of problem solving test as much as 5 items has been given to public junior high school (PJHS) 3 from upper level school in Bandung, which was followed by 33 students of class IX. The average scores for the test is only 19%, (Minarni, 2011).

The students' lack of mathematical problem solving ability has attracted the attention of many education researchers in all over the world. Most researchers find that it is mathematical prior knowledge which is responsible to the students' difficulty in solving mathematical problems, others found that it is the students' unskilled in selecting and applying their knowledge to complete the task as the cause of students' difficulty in solving mathematical problems.

To overcome the problem of the students' acquisition of MUA and MPSA at 8th grade in Bandung, the author conducted the research which title 'The Effect of Problem-based Learning towards MUA and MPSA of the students at 8th grade PJHS 3'.

Problem-based learning (PBL) is chosen in this research because direct instruction or conventional learning approach could not promote students' mathematical problem solving ability as hypothesized by Ronis (2008). In direct instruction, there is no opportunity for the students to promote their ability in solving mathematical problems. Meanwhile, in problem-based learning (PBL) class, students work with classmates to solve complex and authentic problems that help develop content knowledge as well as problem-solving, reasoning, communication, and self-assessment skills. These problems also help to maintain student interest in material because students realize that they are learning the skills needed to be successful in the field.

The content and structure of PBL may differ, but it can be inferred from Arends (2004) that the general goals and learning objectives tend to be similar; PBL begins with the assumption that learning is an active, integrated, and constructive process influenced by social and contextual factors. It can be hoped that PBL could improve students' ability in mathematical understanding and mathematical problem solving since PBL starts the learning process by giving problems to the students. PBL has some other characteristics such as the problems should be designed as ill-structured problems; trigger

learning; student-centered; teacher takes role as tutor, coach, or facilitator; students work in groups to analyze, investigate, discuss, and propose solution; and improving and encouraging students' self and group-assessment.

The first step of the research is to design learning instruction, such as learning material, lesson plan, a set of problem to be solved by the students, and a set of test for the purpose of measuring the ability of understanding mathematical problems (MUA) and mathematical problem-solving abilities (MPSA) that will be given at the end of learning process. All of learning material is design based on PBL principle.

A set of test is tried out by the students at grade eight that have the same characteristics with the sample chosen for the research. This step is done in order to get the reliability and validity of the test and to get the current facts of students' capability in solving mathematical problems. Student achievement (performance) in both types of these tests were analyzed and reported in this article as well as the cause of various mistakes the students made in solving mathematical problems.

B. THEORITICAL FRAMEWORK

Talking about comprehension or understanding, Skemp (1976) categorize understanding into two types; relational and instrumental. He described relational understanding as "knowing both what to do and why", and the process of learning relational mathematics as "building up a conceptual structure". The second type is instrumental understanding that he described as "rules without reasons". Nickerson (1985) examining understanding as: an example agreement with experts, being able to see deeper characteristics of a concept, look for specific information in a situation more quickly, being able to represent situations, and envisioning a situation using mental models. However, he also proposed that "understanding in everyday life is enhanced by the ability to build bridges between one conceptual domain and another.

Nickerson (1985) highlighted the importance of knowledge and of relating knowledge: 'The more one know about the subject, the better one understand it. The richer the conceptual context in which one can embed a new fact, the more one can be said to understand the fact.'

Hiebert & Carpenter (1992) specifically defined mathematical understanding as involving the building up of the conceptual 'context' or 'structure' mentioned above. Mathematics is understood if its mental representation is part of a network of representations. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with strong and more numerous connections."

Another important issue that emerges from the above discussion is whether we are referring to understanding as an action or as a result of an action. Sierpiska (1994) clarified this by putting forward three different ways of looking at understanding. First of all, there is the '*act of understanding*' which is the mental experience associated with linking what is to be understood with the 'basis' for that understanding. Examples: mental representations, mental models, and memories of past experiences. Secondly, 'understanding' which is acquired as a result of the acts of understanding. Thirdly, there are the 'processes of understanding' which involve links being made between acts of understanding through reasoning processes, including developing explanations, learning by example, linking to previous knowledge, linking to figures of speech and carrying out practical and intellectual activities. Sierpiska (1994) saw the process of understanding as cognitive activity that takes place over longer periods of time. In making links between understanding of a mathematical concept through reasoning, for example showing why 12×9 gives the same answer as 9×12 , we further develop our understanding of the concept.

Drawing the various view points from past studies together therefore, the definitions of understanding used in this prior research are:

- a. To understand mathematics is to make connections among mental representations of a mathematical concept.
- b. Understanding is the resulting network of representations associated with mathematical concept.

These definitions draw together the idea of understanding being a network of internalized concepts with the clarification of understanding as an action and a result of an action. We have drawn on the definition of Hiebert & Carpenter (1992) and broadly termed what we link with in this network (including the mental representation of what we are trying to understand, mental models and the memories of past experiences) as mental representations.

Another important terminology in this article is MPS (mathematical problem solving). Mathematical problem is a problem that is amenable to being represented, analyzed, and possibly solved, with the methods of mathematics. This can be a real-world problem, such as computing the orbits of the planets in the solar system, or a problem of a more abstract nature, such as Hilbert's problems. It can also be a problem referring to the nature of mathematics itself, such as Russel's Paradox.

Two types of mathematical problem are well known, the first one is known as real-world mathematical problems, these are questions related to a concrete setting, such as

"Rizal has 9 candies. He gives some candies to his friend so only 6 for himself. How many has he give?". Such questions are usually more difficult to solve than regular mathematical exercises like "9– 6", even if one knows the mathematics required to solve the problem. Known as words problems (story problems), they are used in mathematics education to teach students to connect real-world situations to the abstract language of mathematics.

In general, to use mathematics for solving a real-world problem, the first step is to construct a mathematical model of the problem. This involves abstraction from the details of the problem, and the model has to be careful design not to lose essential aspects in translating the original problem into a mathematical one. After the problem has been solved in the world of mathematics, the solution must be translated back into the context of the original problem.

The second one is abstract (pure) mathematical problems which is arose in all fields of mathematics. While mathematicians usually study them for their own sake, by doing so results may be obtained that is fit in application outside the realm of mathematics. Many abstract problems can be solved routinely; others have been solved with great effort; some have unsolved yet until now.

On the other hand, Lester (Branca, 1980) & Halmos (NCTM, 2000) stated that problem solving is the heart of mathematic because various concepts, principles, and procedures are searched for and used in problem solving. From this point of view, problem solving can be called as doing mathematics. The product of problem solving is various of mathematical objects.

Schoenfeld (1992) proposed four categories of knowledge/skills to be mastered so that one can be success in doing mathematics: (1) Resources-proposition and procedural knowledge of mathematics, (2) heuristics - strategies and techniques for problem solving such as working backwards, or drawing figures, (3) control-decisions about when and what resources and strategies to use, and (4) beliefs-a mathematical "world view" that determines how someone approaches a problem. Successful solution of mathematics problems depends upon a combination of resource knowledge, heuristics, control processes and belief, all of which must be learned and taught.

Based on explanation of mathematical understanding and mathematical problem solving, it seems clear that it is important both mathematical process owned by the students, especially for the students in the middle level school (junior high school). With regard to the importance of the student as the ability of understanding and mathematical problem solving, the testis given to measure students' mathematical understanding and

problemsolving ability. The test is given to 8th grade students at public junior high school 3 in Bandung in 2011. The result of this prior research become a basis for the research in developing students' understanding and problem solving ability conducted in 2012 and 2015 (reported in separated paper).

C. METHODOLOGY

1. Population

The population of the research is public junior high school (PJHS) students in Bandung. There are 52 PJHS in Bandung. Each school consist of more then six class for each grade. Each class comprise more than 30 students.

2. Sample

Sample size is 33 students from class A and 33 students from class B taken from grade IX PJHS 3 Bandung which is belong to upper level school.

3. Instrumentation

The instrument used to collect the data is a set of mathematical understanding test and a set of mathematical problem solving test. The aspect used to design mathematical understanding test comprises:

- a. Represent the problem into diagram, graph, or table
- b. Create mathematical model
- c. Give example or nonexample for mathematical concept
- d. Classified an example into mathematical concept
- e. Knowing the pattern of the problems
- f. Knowing similarities and differences of the concept
- g. Infering the solution of the problem

The aspect used to design mathematical problem solving test consist of:

- (1) Make mathematical equation (mathematical model) from the problem
- (2) Choose and applied appropriate strategies to solve the problems
- (3) Check and articulate the solution

One of seven item of mathematical understanding test and one of six items of MPS test are presented in part D of this article along with students' answer for this item test. Other items of the test are available at the researcher.

D. RESULTS OF THE RESEARCH

Test that have been design in relation to MUA (mathematical understanding ability) and MPS (mathematical problem solving) is tried out at PJHS 3 Bandung in September 2011.

1. Students Achievement in MUA and MPSA Test

The students' score in solving mathematical problem test related to mathematical understanding ability (MUA) is presented in Table 1.

Table 1. Students Achievement in Mathematical Understanding Ability (MUA) Test

Subject	Score	Subject	Score	Subject	Score
1	1	12	4	23	4
2	5	13	27	24	14
3	17	14	20	25	14
4	14	15	3	26	11
5	9	16	8	27	6
6	12	17	19	28	19
7	9	18	8	29	17
8	13	19	21	30	11
9	7	20	9	31	11
10	10	21	0	32	18
11	14	22	9	33	5

Ideal score = 28

The students' score for mathematical problem solving ability (MPSA) test is presented in Table 2.

Table 2. Students Achievement in Mathematical Problem Solving ability (MPSA) Test

Subject	Score	Subject	Score	Subject	Score
1	0	12	3	23	16
2	2	13	17	24	11
3	9	14	13	25	7
4	11	15	10	26	0
5	8	16	19	27	9
6	6	17	9	28	2
7	8	18	1	29	9
8	7	19	17	30	3
9	8	20	6	31	2
10	1	21	4	32	13
11	3	22	10	33	3

Ideal score = 24

Mean and percentage of the students' score of MUA and MPS test are presented in Table 3.

Table 3. Statistics of The Students' MUA and MPS

Ability	N	Mean	% of Mean Score	Std. Deviation	Std Error Mean
MUA	33	11,18	39,93	6,267	1,091
MPS	33	7,79	32,46	5,260	,916

Ideal score: MUA = 28, MPS = 24

It can be seen at Table 3, the average score of the students in MUA test is 11,18 (ideal score is 28), while the average score of the students in MPS test is 7,79 (ideal score is 24). It mean, the average achievement of the students in mathematical understanding ability and mathematical problem solving ability are respectively 39,93%, and 32,46%, even not attain 50%.

2. Analysis of The Students Performance for The Test

In this section we analyzed showed that the students' difficulties in solving mathematical problems as well as analyzed the cause of students' mistake or error they did in solving the problem. Table 4 shows the distribution of the number of students on each question and each acquisition test score.

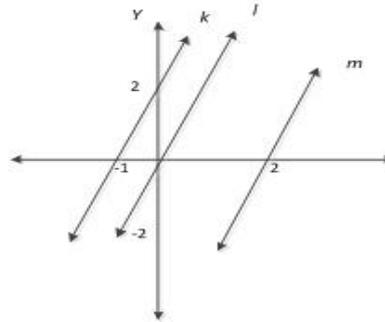
Table 4. Number of students on each question and each score for MUA Test

Item of the test	Score					Sample size
	0	1	2	3	4	
1	1	2	13	4	13	33
2	16	0	2	1	14	33
3	2	2	7	4	18	33
4	19	4	4	2	4	33
5	22	4	3	3	1	33
6	15	2	8	3	5	33
7	19	5	0	2	7	33

It can be seen in Table 3, the majority of students have difficulty in solving problem numbers 5 of MUA test (22 students get zero score), also for number 2, 4, 6 and 7. For these questions, more than 50 % students get score not more than 2. Item number 6 of MUA test presented below. The students give more interesting solution for this problem then for other items. Other items of the test are available at the researcher.

Problem 6

Look at the picture. Line k has the equation $y = 2x + 2$, while $l = 2x$ represent the equation of line l . If line m is parallel to k as well as to l , then find the equation of line m .



The students of 8th grade junior high school should have no difficulty in determining the equation of a straight line if the line is passed a point and there are similarities with other lines parallel to the line requested. Problem 6 want to reveal the students' knowledge of the properties of gradient of two parallel lines and determine the equation of the line after determining its slope. The students often mistakenly choose between the properties of the gradient of two parallel lines and the properties of gradient of the two lines that intersect perpendicularly. The mistakes made by the students because of students' lack of understanding of the concept of gradient, and its because of the concept of gradient is not stored and tightly connected in cognitive structure of the students as stated by Hiebert & Carpenter (1992). One of the students' performance for Problem 6 of MUA test presented below.

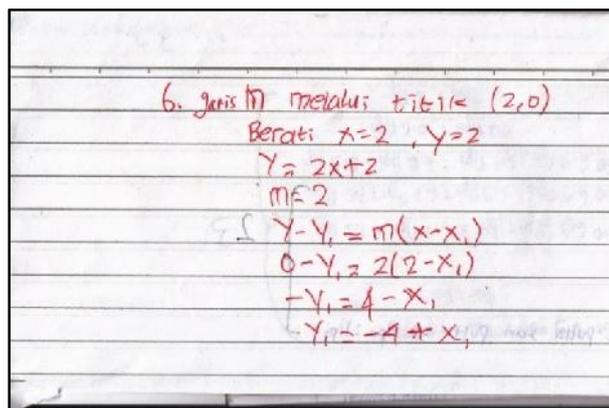


Figure 2. Student' performance for Problem 6 of MUA test

We can see from Figure 2 that the student know how to determine a straight line equation, know how to substitute x_1 and y_1 . But, he fail to execute problem of algebraic equation, so he could not attain the right solution.

After finishing in reviewing all of the students' answer sheet for MUA test, the researcher found that a lot of students could not finish in solving problems due to the students' influency in manipulating algebraic equation.

Table 5. Number of students on each question and each score for MPS Test

Item of the test	Score					Sample size
	0	1	2	3	4	
1	7	12	6	1	7	33
2	17	4	3	0	9	33
3	8	14	3	3	5	33
4	19	6	2	3	3	33
5	28	1	1	1	2	33
6	14	6	1	2	10	33

It can be seen from Table 5 that 50 % of students had difficulty in solving MPS problems numbers 2 , 4 , 5 , and 6. The achievement of most of the students not more than 25 % . Majority students have difficulty in doing item 5 of the test (28 students get zero score, only 2 students get perfect score). This item intended to measuring students' skills in creating a straight line equation of the line. The students have difficulties in creating an equation for a problem because they hard to recall or retrieve their knowledge about the concept of straight line equation from their cognitive structure. Problem number 5 of MPSA test presented below. Other items of the test are available at the researcher.

Problem 5

Line l together with x -axis and y -axes forms triangle which has area 10 unit. If the line passes through $P(2,0)$, then find the equation of the line.

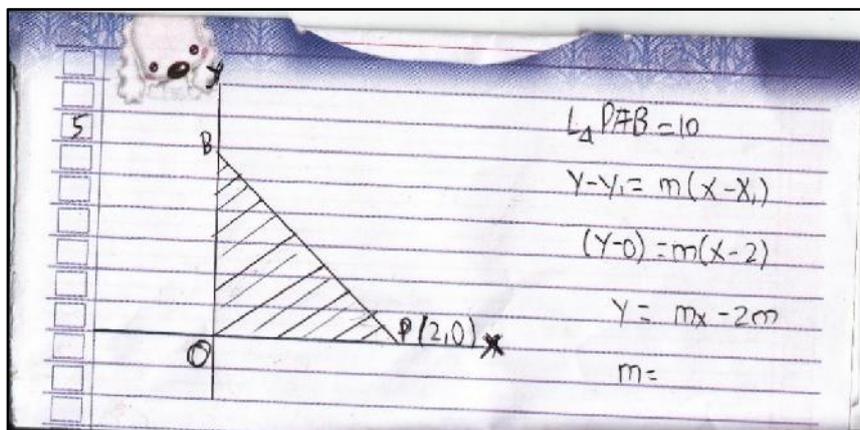


Figure 3. Student' performance for Problem 5 of MPSA test

The students' answers to problem 5 of MUA shows that the students already know how to determine the equation of a line, but then the student only use the coordinates of point P which is passed by line k, whereas for determining the gradient required two points. This student has not yet skilled in determining a gradient of straight line equation. Researchers speculate that these students are less serious about engaging in group-work, although teachers constantly encourage and motivate them. But the real cause for the mistake is because of the students' lack of mathematical prior knowledge.

In general, the result of analyses to all of items of the test showed that the students are lack of specification knowledge or prior knowledge (for example, student's performance in item 5 of MUA test), and this may be due to students are not given the opportunity to build significant knowledge and experience as well as do their own problem-solving in the classroom or in the form of home assignments, students are not stimulated and given the opportunity to solidify and expand their knowledge as recommended by NCTM (2000). And then, a number of the students are not skilled creating mathematical equations, for example when facing item 6 of MPS test. In addition, many students do not understand the problem and is unable to associate their knowledge to solve the problem or merely make representations such as required in problem 6 of MPS test. The students also often make mistakes in doing algebraic operation.

E. Conclusion

The research results showed that:

1. Related to the students' achievement: the students' average score for MUA test is 39,95% and 32,46% for MPS test.
2. Related to some mistakes students made in solving mathematical problems are due to:
 - (a) students' fluency in algebraic operation.
 - (b) lack of mathematical prior knowledge.
 - (c) limited mathematical understanding and representation.

F. Suggestion

It is suggested that the teachers:

1. Give more attention to improve students' capability in solving algebraic problems.
2. Remind and reinforce students' mathematical prior knowledge periodically.
3. Develop students' mathematical understanding & representation ability.

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