FORWARD CLIQUET OPTION FOR GUARANTEED INDEX-LINKED LIFE INSURANCE

Eri Widyastuti

Department of Mathematic, State University of Medan, Indonesia
E-mail: widyaoke@gmail.com

Abstract

The market risk contained in guaranteed index-linked life insurance products can be managed with options. To prevent premature cancellation, a profit structure is chosen which corresponds to forward cliquet options. The pricing of these options is computed for a variety of payoff patterns. When the profit from the index contributes additively to the benefits, then a closed form is possible for prices.

Keywords : guaranteed index-linked life insurance, forward cliquet options

A. INTRODUCTION

The distinguishing feature of the guaranteed index-link life insurance with a traditional life insurance policy is that the benefit obligation at maturity depends on the market value of some reference portfolio. While in traditional life insurance policies provide benefits remain, or policies that may participate in the benefits of loosely associated with the performance of the investment and mortality experience of the insurer. Thus policy-guaranteed life insurance index imposes on policyholders full investment risk, ie the risk that the] equity-linked life products with a certain "lock in" feature in the equity-profit part. We shall derive prices for the corresponding options under the Black and Scholes model with constant interest rate.

B. FORWARD CLIQUET OPTION

Cliquet options are essentially a series of forward-starting *at the money* options with a single premium determined up front, that locks in any gains on specific dates. Floors and caps are then added to fix the minimum and maximum returns. Hence, by construction, cliquet options protect against downside risk, yet they are affordably priced, since the payoff is locally capped, hence never too extreme (e.g. [13]). Payoff from cliquet call option at maturity $T = t_n$

$$\max\left\{\left(S_{t_0}, S_{t_1}, \dots, S_{t_n}\right) - S_{t_0}\right\} \qquad (2.1)$$

and Payoff from cliquet put option at maturity $T = t_n$

$$\max\left\{S_{t_0} - \left(S_{t_0}, S_{t_1}, \dots, S_{t_n}\right)\right\} \qquad (2.2)$$

(e.g. [12]**).**

The profit from European call option which at maturity pays

$$B = K\left(\frac{S(T)}{S(0)} - 1\right)^{+} \qquad (1)$$

K is the fixed sum which participates in the index. And for payoff pattern from sequence of T forward options on $S(t)$ which are *at the money* $(x=s)$

$$B=K\sum_{t=1}^{T}\left(\frac{S(t)}{S(t-1)}-1\right)^{+} \qquad (2)$$

For the cases in which the annual profits bear interest or are compounded, we assume that the interest rate *r* is nonrandom and constant. The payoff at maturity in the case with interest rate *r* equals:

$$B=K\sum_{t=1}^{T}\left(\frac{S(t)}{S(t-1)}-1\right)^{+}(1+r)^{T-t} \qquad (3)$$

If profit is compounded, i.e. if *K* is adjusted each year such that all earlier profits are included, then the payoff at expiration :

$$\begin{aligned} B&=K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)},1\right)-K \\ &=Ke^{\left(\sum_{t=1}^{T}(\log(S(t))-\log(S(t-1)))^{+}\right)}-K \qquad (4) \\ &=K\left(e^{\left(\sum_{t=1}^{T}\left(\log\frac{S(t)}{S(t-1)}\right)^{+}\right)}-1\right) \end{aligned}$$

A payoff yielding a minimal interest rate of r_0 which might be called compounded with floor is the following:

$$B=K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)},1+r_0\right)-K \qquad (5)$$

C. PRICING OF FORWARD CLIQUET OPTION

Pricing of the forward cliquet options under the different payoff patterns under the Black and Scholes model for the index:
$dS(t)=«S(t)dt+\pm S(t)dW(t),0\le t\le T,S(0)=s_0>0$. Here, $W(t),0\le t\le T$, is a standard Wiener process. The pricing of a contingent claim paying an amount *C* at *T* which depends on $S(t),0\le t\le T$, is then done using this equivalent martingale measure P. The price for *C* equals

$$C=e^{(-¢T)}\hat{\mathrm{E}}B$$

The Option Price for Single Premium Payment with Additive Benefit

We defined that :

$$C_{BS}(s,x,{}^2)=s\Phi\left(\frac{\log\left(s/x\right)+\left(¢+\pm^2/2\right)^2}{\pm\sqrt{2}}\right)-e^{(-¢^2)}x\Phi\left(\frac{\log\left(s/x\right)+\left(¢-\pm^2/2\right)^2}{\pm\sqrt{2}}\right)$$

$$C_{BS}(s,{}^2)=s\Phi\left(\frac{\left(¢+\pm^2/2\right)\sqrt{2}}{\pm}\right)-e^{(-¢^2)}s\Phi\left(\frac{\left(¢-\pm^2/2\right)\sqrt{2}}{\pm}\right)$$

where $C_{BS}(s,x,²)$ is the classical Black and Scholes price for a European call with exercise price x, when the underlying has value s, and time to maturity is ², and $C_{BS}(s,²)$ is the value at the money (i.e. $x=s$), with $\Phi(x)=\frac{1}{\sqrt{2®}}\int_{-\infty}^{x} e^{\left(\frac{-t^2}{2}\right)}dt$ the standard normal distribution function. Now, we will derive the options price for the fifth payoff.

For $B=K\left(\frac{S(t)}{S(0)}-1\right)^{+}$, the price of forward cliquet option that corresponding with payoff (1) is:

$$
\begin{aligned}
C &= e^{(-¢T)}\hat{E}B \\
&= e^{(-¢T)}\hat{E}\left(K\left(\frac{S(t)}{S(0)}-1\right)\right)^{+} \\
&= Ke^{(-¢T)}\hat{E}\left(\frac{S(t)}{S(0)}-1\right)^{+} \\
&= Ke^{(-¢T)}\left[\hat{E}\left(\frac{S(t)}{S(0)}\right)-1\right] \\
&= Ke^{(-¢T)}\left[\hat{E}\left(\frac{s_0\exp\left(¢-\frac{\pm^2}{2}\right)t+\pm\sqrt{t}Z}{s_0}\right)-1\right] \\
&= Ke^{(-¢T)}\hat{E}\left[I\left\{\exp\left(¢-\frac{\pm^2}{2}\right)t+\pm\sqrt{t}Z\right\}\right]-Ke^{(-¢T)}\hat{E}[I]
\end{aligned}
$$

We will try to find solution from $\hat{E}\left[I\left\{\exp\left(¢-\frac{\pm^2}{2}\right)T+\pm\sqrt{T}Z\right\}\right]$, with assumed that $c=\pm\sqrt{T}-\mu$

$$
\begin{aligned}
\hat{E}\left[I\left\{\exp\left(¢-\frac{\pm^2}{2}\right)T+\pm\sqrt{T}Z\right\}\right] &= \int_{c}^{\infty} e^{\left\{\left(¢-\frac{\pm^2}{2}\right)T+\pm\sqrt{T}x\right\}}\frac{1}{\sqrt{2®}}e^{-\frac{x^2}{2}}dx \\
&= \frac{1}{\sqrt{2®}}e^{\left(¢-\frac{\pm^2}{2}\right)T}\int_{c}^{\infty} e^{\left\{-\frac{\left(x^2-2\pm\sqrt{T}x\right)}{2}\right\}}dx \\
&= \frac{1}{\sqrt{2®}}e^{¢T}\int_{c}^{\infty} e^{\left\{-\frac{\left(x-\pm\sqrt{T}\right)^2}{2}\right\}}dx \\
&= e^{¢T}\frac{1}{\sqrt{2®}}\int_{-\mu}^{\infty} e^{\left\{-\frac{y^2}{2}\right\}}dx \qquad ;y=x-\pm \\
&= e^{¢T}P\{Z>-\mu\} \\
&= e^{¢T}\Phi(\mu)
\end{aligned}
$$

and $\hat{\mathrm{E}}[I]=\Phi\left(\mu-\pm\sqrt{T}\right)$, then

$$C=Ke^{(-¢T)}\hat{\mathrm{E}}\left[I\left\{\exp\left(¢-\frac{\pm^{2}}{2}\right)T+\pm\sqrt{T}Z\right\}\right]-Ke^{(-¢T)}\hat{\mathrm{E}}[I]$$

$$=Ke^{-¢T}e^{¢T}\Phi(\mu)-Ke^{-¢T}\Phi\left(\mu-\pm\sqrt{T}\right)$$

$$=K\left(\Phi(\mu)-e^{-¢T}\Phi\left(\mu-\pm\sqrt{T}\right)\right)$$

$$=KC_{BS}(1,T)$$

For payoff (1), we obtain the price :

$$C=KC_{BS}(1,T)$$

For payoff (2) $B=K\sum_{t=1}^{T}\left(\frac{S(t)}{S(t-1)}-1\right)^{+}$ the price is:

$$C=e^{(-¢T)}EB$$

$$=\mathrm{e}^{(-¢T)}E\left(K\sum_{t=1}^{T}\left(\frac{S(t)}{S(t-1)}-1\right)^{+}\right)$$

$$=K\,\mathrm{e}^{(-¢T)}\sum_{t=1}^{T}E\left(\frac{S(t)}{S(t-1)}-1\right)^{+}$$

Where $S(t)=S_0e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)t+\pm W(t)\right]}$ and $S(t-1)=S_0e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)(t-1)+\pm W(t-1)\right]}$

$$=S_0e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)t+\pm W(t)-¢+\frac{1}{2}\pm^{2}-\pm W(1)\right]}$$

Then

$$\frac{S(t)}{S(t-1)}=\frac{S_0e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)t+\pm\hat{W}(t)\right]}}{S_0e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)t+\pm\hat{W}(t)-¢+\frac{1}{2}\pm^{2}-\pm\hat{W}(1)\right]}}$$

$$=e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)+\pm\hat{W}(1)\right]}$$

$$=e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)+\pm\sqrt{1}Z\right]}$$

Then

$$C=Ke^{-¢T}\sum_{t=1}^{T}\hat{\mathrm{E}}\left(I\left(\mathrm{e}^{\left(\left(¢-\frac{1}{2}\pm^{2}\right)+\pm\sqrt{1}Z\right)}\right)-1\right)^{+}$$

$$=Ke^{-¢T}\sum_{t=1}^{T}\hat{\mathrm{E}}\left(I\left(\mathrm{e}^{\left(¢-\frac{1}{2}\pm^{2}+\pm\sqrt{1}Z\right)}\right)-\hat{\mathrm{E}}[I]\right)$$

We will try to find solution from $\hat{\mathrm{E}}\left[I\left\{\exp\left(¢-\frac{\pm^{2}}{2}\right)+\pm\sqrt{1}Z\right\}\right]$,with ssumed $c=\pm\sqrt{1}-\mu$

$$\hat{\text{E}}\left[I\left\{\exp\left(¢-\frac{±^2}{2}\right)+±\sqrt{1}Z\right\}\right]=\int_c^{\infty} e^{\left\{\left(¢-\frac{±^2}{2}\right)+±\sqrt{1}x\right\}}\frac{1}{\sqrt{2®}}e^{-\frac{x^2}{2}}dx$$

$$=\frac{1}{\sqrt{2®}}e^{\left(¢-\frac{±^2}{2}\right)}\int_c^{\infty} e^{\left\{-\frac{\left(x^2-2±x\right)}{2}\right\}}dx$$

$$=\frac{1}{\sqrt{2®}}e^{¢}\int_c^{\infty} e^{\left\{-\frac{(x-±)^2}{2}\right\}}dx$$

$$=e^{¢}\frac{1}{\sqrt{2®}}\int_{-µ}^{\infty} e^{\left\{-\frac{y^2}{2}\right\}}dx \qquad ;y=x-±$$

$$=e^{¢}P\{Z>-µ\}$$

$$=e^{¢}\Phi(µ)$$

Then

$$C=Ke^{-¢T}\sum_{t=1}^{T}\hat{\text{E}}\left(I\left(\text{e}^{\left(\left(¢-\frac{1}{2}±^2\right)+±\sqrt{1}Z\right)}\right)-1\right)^+$$

$$=Ke^{-¢T}\sum_{t=1}^{T}\hat{\text{E}}\left(I\left(\text{e}^{\left(¢-\frac{1}{2}±^2+±\sqrt{1}Z\right)}\right)-\hat{\text{E}}[I]\right)$$

$$=Ke^{-¢T}\left[\sum_{t=1}^{T}\left(e^{¢}\Phi(µ)-\Phi\left(µ-±\sqrt{1}\right)\right)\right]$$

$$=Ke^{-¢T}\left[\sum_{t=1}^{T}e^{¢}\left(\Phi(µ)-e^{-¢}\Phi\left(µ-±\sqrt{1}\right)\right)\right]$$

$$=Ke^{-¢T}\sum_{t=1}^{T}e^{¢}C_{BS}(1,1)$$

$$=Ke^{(-T+1)¢}TC_{BS}(1,1)$$

$$=K(1+r)^{(-T+1)}TC_{BS}(1,1)$$

For payoff (3) we obtain the following price:

$$C=\text{e}^{(-¢T)}\,\hat{\text{E}}K\sum_{t=1}^{T}\left(\frac{S(t)}{S(t-1)}-1\right)^+(1+r)^{T-t}$$

$$=K\sum_{t=1}^{T}(1+r)^{-T}(1+r)^{T-t}\,\hat{\text{E}}\left(\frac{S(t)}{S(t-1)}-1\right)^+$$

$$=K\sum_{t=1}^{T}(1+r)^{-t}\,\hat{\text{E}}\left(\text{e}^{\left(\left(¢-\frac{1}{2}±^2\right)+±W(1)\right)}-1\right)^+$$

$$=K\sum_{t=1}^{T}(1+r)^{-t}e^{¢}C_{BS}(1,1)$$

$$= K\sum_{t=1}^{T}(1+r)^{-t}(1+r)C_{BS}\left(1,1\right)$$

$$= K\sum_{t=1}^{T}(1+r)^{-t+1}C_{BS}\left(1,1\right)$$

$$= K\sum_{t=0}^{T}\left(1+r\right)^{-t}C_{BS}\left(1,1\right)$$

$$= K\frac{1-(1+r)^{-T}}{1-\frac{1}{1+r}}C_{BS}(1,1)$$

$$= K\left(1-\left(1+r\right)^{-T}\right)\frac{1+r}{r}C_{BS}\left(1,1\right)$$

Also for payoff (4) we obtain a closed form for the price. The exponent in the payoff

$$\sum_{t=1}^{T}\left(\log\left(S(t)\right)-\log\left(S(t-1)\right)\right)^{+}=\sum_{t=1}^{T}\left[\log e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)t+\pm W(t)\right]}-\log e^{\left[\left(¢-\frac{1}{2}\pm^{2}\right)(t-1)+\pm W(t-1)\right]}\right]^{+}$$

$$=\sum_{t=1}^{T}\left[\left(¢-\frac{1}{2}\pm^{2}\right)(t)+\pm W(t)-\left(¢-\frac{1}{2}\pm^{2}\right)(t-1)+\pm W(t-1)\right]^{+}$$ is the sum of

$$=\sum_{t=1}^{T}\left(¢-\frac{1}{2}\pm^{2}-\pm\left(W(t)-W(t-1)\right)\right)^{+}$$

is the sum of iid random variables, and therefore

$$C=\mathrm{e}^{(-¢T)}\,\hat{\mathrm{E}}\left[K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)},1\right)-K\right]$$

$$=\mathrm{e}^{(-¢T)}\,K\left(\hat{\mathrm{E}}\left(\mathrm{e}^{\sum_{t=1}^{T}\log S(t)-\log S(t-1)}-1\right)\right)$$

$$=e^{(-¢T)}K\left(\hat{\mathrm{E}}\left(e^{\sum_{t=1}^{T}\left(¢-\frac{1}{2}\pm^{2}-\pm\left(W(t)-W(t-1)\right)\right)^{+}}\right)-1\right)$$

$$=e^{(-¢T)}K\left(\hat{\mathrm{E}}\left(e^{\sum_{t=1}^{T}\left(¢-\frac{1}{2}\pm^{2}+\pm W(1)\right)^{+}}\right)-1\right)$$

$$=e^{(-¢T)}K\left(\hat{\mathrm{E}}e^{\sum_{t=1}^{T}\left(¢-\frac{1}{2}\pm^{2}+\pm W(1)\right)^{+}}\right)-e^{(-¢T)}K$$

$$= Ke^{-¢T}\left(\hat{\text{E}}e^{\left(¢-\frac{1}{2}\pm^{2}+\pm W(1)\right)^{+}}\right)^{T} - Ke^{-¢T}$$

$$= K\left(\text{e}^{(-¢)}\,\hat{\text{E}}\,\text{e}^{\left(¢-\frac{1}{2}\pm^{2}+\pm W(1)\right)^{+}}\right)^{T} - Ke^{-¢T}$$

With $S = ¢ - \frac{1}{2}\pm^{2} + \pm W(1)$, we obtain

This yields the price

$$C = K\left(e^{-¢}\left(1+e^{¢}C_{BS}(1,1)\right)\right)^{T} - Ke^{-¢T}$$
$$= K\left(e^{-¢} + C_{BS}(1,1)\right)^{T} - Ke^{-¢T}$$
$$= K\left(\frac{1}{1+r} + C_{BS}(1,1)\right)^{T} - K(1+r)^{-T}$$

$$\hat{\text{E}}\,\text{e}^{(S)^{+}} = \hat{\text{E}}\,\text{e}^{(S)1_{(S>0)}} + P(S \le 0)$$
$$= \hat{\text{E}}\left(\text{e}^{(S)}-1\right)1_{(S>0)} + 1$$
$$= \hat{\text{E}}\left[I\left\{e^{(S)}-1\right\}\right] + 1$$
$$= \hat{\text{E}}\left[I\left(e^{(S)}\right)\right] - \hat{\text{E}}[I] + 1$$
$$= \left(e^{¢}\Phi(\mu) - \Phi(\mu-\pm)\right) + 1$$
$$= \left(e^{¢}\left(\Phi(\mu) - e^{-¢}\Phi(\mu-\pm)\right)\right) + 1$$
$$= 1 + \text{e}^{¢}\,C_{BS}(1,1)$$

for the corresponding option with payoff pattern (4). Similarly, for payoff (5) $B = K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)}, 1+r_0\right) - K$, we obtain the option price

$$C = e^{-¢T}\left[K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)}, 1+r_0\right) - K\right]$$
$$= e^{-¢T}K\prod_{t=1}^{T}\max\left(\frac{S(t)}{S(t-1)}, 1+r_0\right) - e^{-¢T}K$$
$$= Ke^{-¢T}\left(\hat{\text{E}}\exp\left(¢ - \frac{\pm^{2}}{2} + \pm W(1)\right)^{+}\right)^{T} - Ke^{-¢T}$$

With $S = ¢ - \frac{1}{2}\pm^2 + \pm W(1)$, we obtain

$$\begin{aligned}
\hat{\mathrm{E}}\,\mathrm{e}^{(S)^+} &= \hat{\mathrm{E}}\left[\mathrm{e}^{(S)(1+r_0)_{(S>0)}}\right] + P(S \le 0) \\
&= \hat{\mathrm{E}}\left[\mathrm{e}^{(S)} - (1+r_0)\right]1_{(S>0)} + (1+r_0) \\
&= \hat{\mathrm{E}}\left[I\left\{e^{(S)} - (1+r_0)\right\}\right] + (1+r_0) \\
&= \hat{\mathrm{E}}\left[I\left(e^{(S)}\right)\right] - (1+r_0)\hat{\mathrm{E}}[I] + (1+r_0) \\
&= \left(e^{¢}\Phi(\mu) - (1+r_0)\Phi(\mu - \pm)\right) + (1+r_0) \\
&= \left(e^{¢}\left(\Phi(\mu) - e^{-¢}(1+r_0)\Phi(\mu - \pm)\right)\right) + (1+r_0) \\
&= (1+r_0) + \mathrm{e}^{¢}\, C_{BS}(1, 1+r_0, 1)
\end{aligned}$$

This yields the price

$$\begin{aligned}
C &= K\left(e^{-¢}\left((1+r_0) + e^{¢}C_{BS}(1, 1+r_0, 1)\right)\right)^T - Ke^{-¢T} \\
&= K\left(\frac{1+r_0}{1+r} + C_{BS}(1, 1+r_0, 1)\right)^T - K(1+r)^{-T}
\end{aligned}$$

D. STUDY CASE

Data that used in this paper to derived the options price from the fifth payoff is based on daily closing prices of historical index DOW, DAX, FTSE, Nasdaq, and S&P500, which run from time 10 May 2008 to time 10 May 2014 (see http://finance.yahoo.com/).

By the standard estimator of volatility

$$\pm = \sqrt{\frac{\sum_{t=1}^{n}\left(R_t - \overline{R}_t\right)^2}{n-1}}$$

using microsoft Exceel software, we find:

Table 4.1. Volatility Of The Index Price

No	Index	Volatility
1	DOW	0,5386
2	DAX	0,6185
3	FTSE	0,5604
4	Nasdaq	0,6166
5	S&P 500	0,1169

Based on United States current interest rate from time 16 May 2008 to time 10 May 2014 accessed from http://www.fxstreet.com/fundamental/interest-rates-table/, is 0,25 %. Then $r = 0,0025$

To find payoff that derived from *Forward-Cliquet Options* for single premium payment with additive benefit and finding the options price, needed some informations; i.e constant interest rate (r), guaranteed interest rate (r_0), participation (K), volatility (±), and time to maturity (T) .

For the index DOW, DAX, FTSE, Nasdaq, and S&P500; where guaranteed interest rate $r_0 = 0,001$ and participation in the index $K = 0,9$; the payoffs are :

Table 4.2. Payoffs For Single Premium Payment

	DOW	DAX	FTSE	Nasdaq	S&P500
Payoff 1	1,1859	1,1473	1,1757	1,1482	1,3701
Payoff 2	2,0387	2,0387	2,0387	2,0387	2,0387
Payoff 3	2,0664	2,0664	2,0664	2,0664	2,0664
Payoff 4	8,2383	8,9683	8,4330	8,9504	5,1143
Payoff 5	8,2522	8,9837	8,4473	8,9658	5,1222

And the options price for each payoff is

Table 4.3. Stock Index Option Price for Dow

Stock Index Option	DOW				
	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5
Year 1	0,1920	0,1920	0,1920	0,1920	0,1925
Year 2	0,2686	0,3830	0,3834	0,4239	0,4252
Year 3	0,3254	0,5730	0,5744	0,7043	0,7066
Year 4	0,3715	0,7621	0,7650	1,0432	1,0471
Year 5	0,4107	0,9502	0,9550	1,4532	1,4590
Year 6	0,4449	1,1375	1,1446	1,9491	1,9576

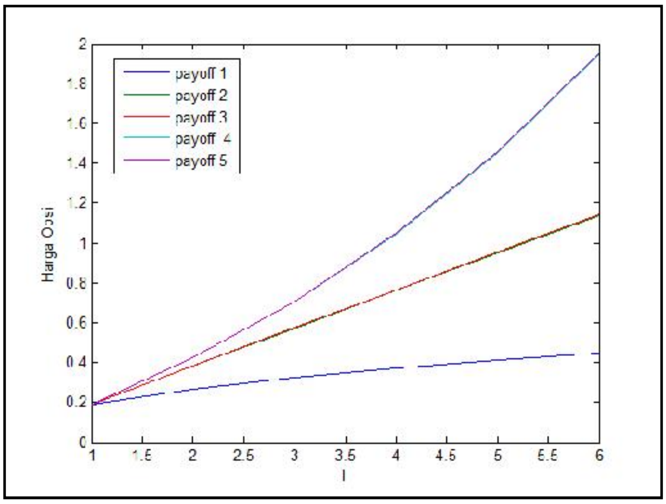

Figure 4.1. Price Option Based on Payoff 1 s / d 5 in stock DOW

Table 4.4. Stock Index Option Price For Dax

Stock Index Option	DAX				
	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5
Year 1	0,2194	0,2194	0,2194	0,2194	0,2200
Year 2	0,3058	0,4378	0,4383	0,4913	0,4927
Year 3	0,3690	0,6550	0,6567	0,8282	0,8307
Year 4	0,4198	0,8712	0,8745	1,2458	1,2501
Year 5	0,4624	1,0863	1,0917	1,7637	1,7703
Year 6	0,4991	1,3003	1,3084	2,4060	2,4159

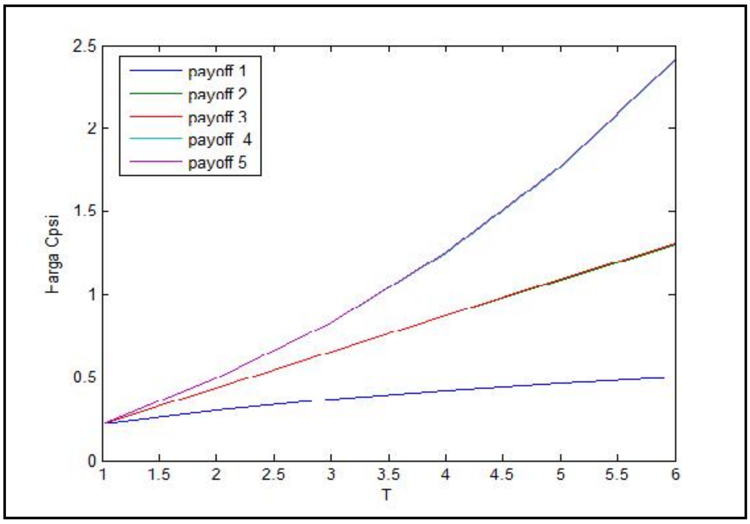

Figure 4.2. Price Option Based on Payoff 1 s / d 5 in stock DAX

Table 4.5. Stock Index Option Price for Ftse

Stock Index Option	FTSE				
	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5
Year 1	0,1995	0,1995	0,1995	0,1995	0,2000
Year 2	0,2788	0,3980	0,3985	0,4422	0,4435
Year 3	0,3374	0,5955	0,5970	0,7376	0,7400
Year 4	0,3849	0,7920	0,7950	1,0972	1,1012
Year 5	0,4251	0,9875	0,9925	1,5352	1,5412
Year 6	0,4601	1,1821	1,1895	2,0687	2,0775

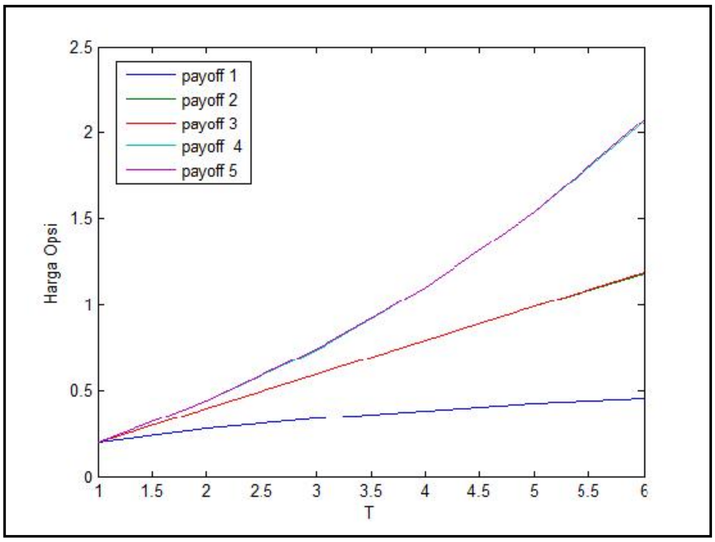

Figure 4.3. Price Option Based on Payoff 1 s / d 5 in stock FTSE

Table 4.6. Stock Index Option Price For Nasdaq

Stock Index Option	NASDAQ				
	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5
Year 1	0,2188	0,2188	0,2188	0,2188	0,2193
Year 2	0,3049	0,4365	0,4370	0,4897	0,4910
Year 3	0,3680	0,6531	0,6547	0,8252	0,8277
Year 4	0,4187	0,8686	0,8719	1,2409	1,2451
Year 5	0,4612	1,0830	1,0885	1,7560	1,7626
Year 6	0,4979	1,2964	1,3045	2,3946	2,4044

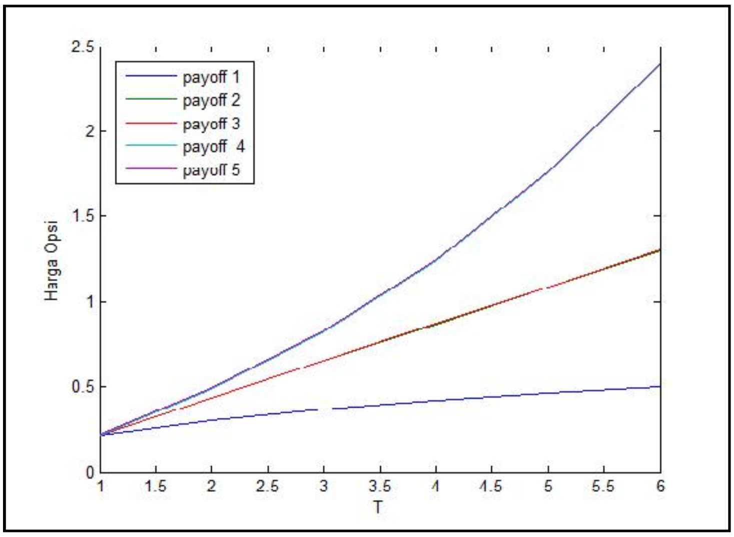

Figure 4.4. Price Option Based on Payoff 1 s / d 5 in stock NASDAQ

Table 4.7. Stock Index Option Price for S&P500

Stock Index Option	S&P500				
	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5
Year 1	0,0430	0,0430	0,0430	0,0430	0,0435
Year 2	0,0614	0,0858	0,0859	0,0879	0,0889
Year 3	0,0757	0,1284	0,1288	0,1347	0,1362
Year 4	0,0879	0,1708	0,1715	0,1835	0,1856
Year 5	0,0987	0,2130	0,2141	0,2344	0,2372
Year 6	0,1085	0,2550	0,2566	0,2875	0,2910

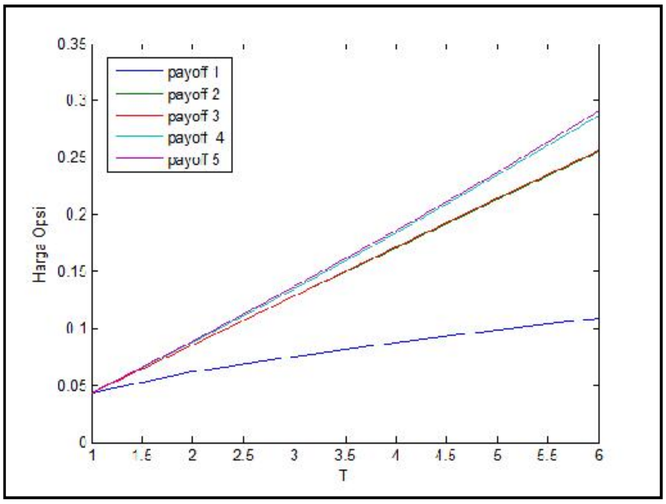

Figure 4.5. Price Option Based on Payoff 1 s / d 5 in stock S&P500

E. CONCLUSIONS

1. Forward cliquet option is one option that is used to anticipate the cancellation of the contract of the policyholder because the market risk inherent in guaranteed index-linked life insurance products.
2. In cliquet forward option, the annual profit lock-in in the index. The profit will not be lost if the losses in the index occur in the future.
3. The price for payoff (1) is not always smaller than the price for the forward cliquet option (2). Even more surprisingly, the price of the forward cliquet option is not increasing in time. This is due to the fact that the profit is not included in the number K.

F. REFERENCES

Bain, J. Lee., and Max Engelhardt, 1992, Introduction to Probability and mathematical statistics, Second Edition, Duxbury Press, California.

Bodie, Alex Kane, Alm J. Maras., 2005, Investments 6 th ed, Mc. Gram Hill Company, Inc

Boyle, Ph., and Schwartz, E., 1977, Equilibrium Pricing of Guarantees under Equity-Linked Contracts, Journal Risk and Insurance 44, 639-660.

Brennan, M., and Schwartz, E., 1976, The Pricing of Equity-Linked Life Insurance policies with an Asset Value Guarantee, Journal of Financial Economics 3, 195-213.

Capinski, M., and Tomasz Zastawniak, 2003, Mathematics for Finance; An Introduction to Financial Engineering, Springer Undergraduate Mathematics, London.

Etheridge, A., 2002, A Course in Financial Calculus, Cambridge University Press.

Harrison, P., 1981, Arbitrage Theory in Continuous Time, Oxpord University Press.

Higham, D. J., 2004, An Introduction to Financial Option Valuation, Cambridge University Press, Cambridge.

Hipp, Christian., 1999, Options for Guaranteed Index-Linked life Insurance, http://arXiv.math, diakses 20 Maret 2011.

Hogg and Craig, 2005, Introduction of Mathematical Statistics, Fourth Edition, Macmillan Publishing Co. Inc., New York.

Hull, J., 2006, Option, Futures, and Other Derivative Securities, Prentice-Hall International, Inc., Toronto.

Jurgen and Wolfgang Karl Hardle, 2011, Statistics of Financial Markets, An Introduction 3 rd Edition, Springer.

N. Patel., 2002, The evolving art of pricing cliquets, Risk 15, no. 7.

Ochi, M.K., 1990, Applied Probability & Stochastic Processes : In Engineering and Physical Science, John Willey & Sons, Inc., Canada.

Ross, Sheldon M, 2003, An Elementary Introduction to Mathematical Finance Second Edition, University of California, Berkeley.

Sahid., 2006, Panduan Praktis MATLAB, Penerbit Andi, Yogyakarta.

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