MS-003

AN IMPROVED APPROACH FOR SOLVING THE PLANT CYCLE LOCATION PROBLEM

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ABSTRACT

The Plant Cycle Location Problem (PCLP) is a combinatorial problem. It can be used in routing and telecommunications. Such problem belongs to an NP-hard problem. In this paper we propose an integer linear programming formulation, and we develop a combination of exact method and heuristic approach for solving the problem.

Key words: *routing, location, integer programming, combination approach*

INTRODUCTION

We present the combination of *Plant-Cycle Location Problem* (PCLP), capacity management, with the well-known *Capacitated Facility Location Problem* (CFLP) (see Cornuejols et al. (1990)). We define the problems as follows. Given two sets of locations, one with customers and another with potential plants (or depots). The travel distance between two locations is assumed to be known and symmetric. The opening of each potential plant has a given known cost. Another important thing from capacity management point of view, each potential plant should have a capacity that limits the number of customers it can serve. As in the CFLP, PCLP consists of choosing the plants to be opened and the assignment of customers to opened plants minimizing the cost. The novelty respect to the CFLP is that in PCLP the customers assigned to a plant must be served with a cycle, thus the cost of a solution also includes the routing cost.

Therefore, the PCLP looks for a set of disjoint cycles covering all the customers, each one containing exactly one plant, and minimizing the total sum of cost of opening the selected plants, cost of customer-plant assignments, and routing cost.

The problem considers capacity constraints limiting the number of customers that can be served by an open plant. In a more general case, a different demand could be associated to each customer and the constraint would limit the total demand served by each plant. Nevertheless, this extension was not present in the application motivating the PCLP.

When there is only one plant and there are no assignment costs, the uncapacitated PCLP reduces to the *Traveling Salesman Problem* (TSP). Moreover, the PCLP reduces to the *Vehicle*

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Routing Problem with unit demand (VRP) when all the plants have the same location. Hence, PCLP is □□□-hard in the strong sense and has several applications in the routing context. See, e.g., Toth and Vigo (2001) for a survey on the CVRP.

Present a 0-1 integer linear programming model, and describe a branch-and-cut algorithm for the exact solution based on several families of inequalities strengthening the LP-relaxation of the model. We also provide computational results showing the good performance of our approach on random instances.

DISCUSSION

Mathematical Model

Let $V = I \cup J$ be the set of locations, where I represents the locations of the customers and J the potential plant locations. Let E be the set of undirected edges linking all possible pairs of locations in V and $G = (V, E)$ the graph on which the PCLP is defined. Each potential plant location $j \in J$ has an associated cost f_j, and can serve at most q_j customers. Associated to each customer $i \in I$ and each potential depot $j \in J$ there is an assignment cost d_{ij}, and associated to each edge $e \in E$ there is a routing cost c_e.

Mathematically, the PCLP can be formulated by defining the following variables. For each plant $j \in J$, let y_j be a binary variable that takes value 1 if the plant j is open in the solution and 0 otherwise. Every edge $e \in E$ has an associated integer variable x_e taking value 2 if one of its extreme vertices is a plant and the other is a customer, and they are the only points in a cycle; value 1 if the edge e is part of a cycle visiting other costumers besides its extremes; and value 0 otherwise. For each $i \in I$ and each $j \in J$ a binary variable z_{ij} takes value 1 if the customer $i \in I$ is assigned to the plant $j \in J$, and value 0 otherwise.

Addition, for each set of vertices $S \subset V$ we define

$$\delta(S) := \{[u; v] \in E : u \in S; v \notin S\},$$

$$E(S) := \{[u; v] \in E : u \in S; v \in S\},$$

And for any $v \in V$ we write $\delta(v)$ instead of $\delta(\{v\})$. Moreover, for each $F \subset E$ we write $x(F)$ instead of $\sum_{e \in F} x_e$. Then the model is the following:

$$\text{minimize} \quad \sum_{j \in J} f_i y_j + \sum_{e \in E} c_e x_e + \sum_{i \in I} \sum_{j \in J} d_{ij} z_{ij} \tag{1}$$

subject to :

$$\sum_{j \in J} z_{ij} = 1 \qquad \text{for all } i \in I, \tag{2}$$

$$\sum_{i \in I} z_{ij} \le q_j y_j \qquad \text{for all } j \in J, \tag{3}$$

$$x(\delta(i)) = 2 \qquad \text{for all } i \in I, \tag{4}$$

$$x(\delta(j)) = 2y_j \qquad \text{for all } j \in J, \tag{5}$$

$x(\delta(S)) \geq 2 \sum_{j \in J \setminus S} z_{ij}$ for all $S \subset V, i \in S \cap I,$ (6)

$x_{ij} \leq 2z_{ij}$ for all $i \in I, j \in J,$ (7)

$x_{ii'} + z_{ij} + z_{i'j'} \leq 2$ for all $i, i' \in I, j, j' \in J,$ (8)

$y_j \in \{0, 1\}$ for all $j \in J,$ (9)

$z_{ij} \in \{0, 1\}$ for all $i \in I, j \in J,$ (10)

$x_{ij} \in \{0, 1, 2\}$ for all $[I, j] \in E.$ (11)

Constraints (2) enforce each customer i to be assigned to exactly one plant j. Capacity constraints (3) limit to q_j the number of customers a plant $j \in J$ can serve, and prevent customers to be assigned to a plant not open. Constraints (4) and (5) are degree constraints, and they ensure that the degree of every customer is 2, and that the degree of every potential plant is 2 if and only if it belongs to a cycle. Constraints (6) are connectivity constraints. They state that each set of vertices $S \subset V$ must be connected to its complement by at least 2 edges whenever there is pair of vertices i and j such that i is a customer in S, j is a plant not in S, and i is assigned to j. Constraints (7) state that if customer i is not assigned to plant j then the edge $[i, j]$ can not be routed. Constraints (8) state that if customers i and i' are assigned to different depots j and j', then they cannot be in the same cycle. Finally, Constraints (9) – (11) are the integrality constraints for the different kind of variables.

Feasible Neighborhood Heuristic Search

While a straightforward brand-and-bound approach could be adopted, for many classes of large-scale problems such a procedure would be prohibitively expensive in terms of total computing time. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed to vary in discrete steps.

This may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as nonbasic.

The procedure may be summarized as follows:

Step 1 : Solve the problem ignoring integrality requirements.

Step 2 : Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution

Step 3 : Divide the set I of integer variables into the set I_1 at their bounds that were nonbasic at the continuous solution and the set I_2, $I = I_1 + I_2$.

Step 4 : Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step5: At the solution obtained in step 4, examine the reduced costs of the variables in I_1.

If any should be released from their bounds, add them to set I_2 and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set $I_{2.}$

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.

In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasics, the integers in the set I_2 must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is include in the problem.

CONCLUSION.

This paper present an integer programming model for PCLP which combine capacity management and CFLP. We address a combined approach for solving the problem.

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