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AN ACTIVE CONSTRAINED BASED APPROACH FOR SOLVING PROBLEMS FOR POSITIONING NEW PRODUCTS UNDER RISK

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ABSTRACT

Currently, manufacturers are faced with the rapid change in technology and customer's preferences. The problem for positioning new products is a marketing problem faced by a firm which wishes to position a new brand product considering customer's preferences. The aim of the problem considered is to optimally design a new product in order to attract the largest number of consumers. This paper addresses a mixed integer nonlinear programming model to formulate the positioning problem. A direct search approach is proposed to solve the model. A computational experience is presented.

Keywords: *Positioning product problem, modeling, Stochastic MINLP, direct search*

INTRODUCTION

Product positioning refers to the positioning of a product in a perceptual attribute space such that it closely matches the consumer perception of the various product attributes (Thakur et al., 2000). For a marketing manager, optimizing a new Product's positioning is a critical and difficult decision. Addressing this issue, Shocker and Srinivasan (1979) developed a framework for identifying optimal new product concepts using joint space models of consumer perceptions and preferences. Joint space analysis entails mapping the locations of existing products and ideal points for each individual (or market segment) use multidimensional scaling (MDS) of consumer perceptions via factor analysis, discriminate analysis or similarity scaling. Using this joint mapping of ideal points and product locations, a manager can model consumers' choices of existing products, predict their responses to new products, and identify optimal new product concepts.

Karadeniz (2009) also emphasizes about the important of product positioning in marketing management. Kwak and Kim (2013) discuss about positioning products in which the market has rapid changes in technology and customer preferences. They propose a mathematical model, in which the model aims to maximize the profit from remanufacturing, given a number of units of end-of-life product.

In the ensuing time period, there have been a number of algorithms developed to identify optimal new product positions from MDS-based maps of consumer perceptions and preferences. Thorough reviews of the MDS-based product positioning literature can be found in

Sudharshan et al. (1987, hereafter SMS), Green and Krieger (1989) and Kaul and Rao (1995). Each step in this evolution was motivated, in part, by attempts to improve the realism of the consumer choice setting. For example, the algorithms that account for a probabilistic choice model tends to provide better solutions, larger share projections, for new product positions (Sudharshan et al., 1987).

In this paper we assume that the consumer first decides his/her budget for buying from a product class. Then the consumer identifies the set of products from the product class that meet his/her budget constraint, evaluate them with the help of a weighted multi-attribute utility model and chooses the product with the highest utility. Therefore we could propose a mixed integer nonlinear programming (MINLP) model to solve the firm's problem of identifying an optimal new product position. The objective is to identify a point in the multi-dimensional attribute space that is closer than the existing product in the product class to the ideal point of as many consumers as possible.

The organization of this paper is as follows. In the next section, we briefly discuss previous research on MDS-based optimal product positioning and building the 'perfect' product. This is followed by a description of the model. The algorithm and results are presented next. We conclude the paper with a discussion of the result.

OPTIMAL POSITIONING LITERATURE REVIEW

In their review, Shocker and Srinivasan (1979) formalized the process of identifying optimal new product concepts using input from consumers at every stage from defining the market to predicting the success of a new product. Since then, a number of algorithms have been developed for MDS-based product positioning. The early approaches (e.g., Albers, 1979; Albers and Brockhoff, 1977; Gavish et al., 1983) had two limitations in common. First, the search methods for these procedures were dependent on the number of ideal points (individuals or segments) in the joint space. Consequently, as the number of ideal points rose, so did the complexity of the optimization problem. Second, these algorithms were formulated for the single choice problem in which the demand from each ideal point is assumed to be completely captured by the closest product to it. In essence, this model suggests a consumer always chooses the product nearest to their ideal. While the first limitation simply slowed down the convergence to a suitable solution, the second limitation ignored empirical evidence about the nature of consumers' choices in many consumer markets.

It has been shown in studies of panel data (beginning with Massy et al., 1970) that consumers often choose probabilistically from a small set of products in the market. One might attribute this behavior to the effects of promotions or availability. However, it has been observed that even if all brands are equally available at no cost, most (53 out of 77) consumers do not choose only their most preferred brand (Best, 1976). This indicates that the probabilistic choice behavior may be a product of variety seeking or factors other than environmental effects (McAlister and Pessemier, 1982).

In 1987, SMS presented a new product positioning algorithm called PRODSRCH which incorporated a probabilistic model of consumer choice. In their formulation, demand from an ideal point is distributed to a product in inverse proportion its relative distance from the ideal point so long as the product is within the fixed size choice set of the ideal point. Otherwise, the product captures no demand share from that ideal point.Kwong et al. (2011) address a new methodology for optimal product positioning by considering engineering constraints. The method is based on perceptual mapping and house quality in order to link the consumer perceptual space, and product engineering space.

To illustrate the differences between the single choice model and the probabilistic choice model, we will use the Shocker and Srinivasan (1974) spatial choice model for finite ideal points. This notation will be used throughout the balance of the paper.

$x_{i,p}$ is the location ith ideal point on the pth dimension,

$y_{j,p}$ is the modal perception of the jth product on the pth dimension,

$w_{i,p}$ is the relative importance of the pth dimension to the ith ideal point,

S_i is the sales potential for ideal point i.

The weighted Euclidean distance ($d_{i,j}$) between the ith ideal point and jth product position is given by Eq. (1).

$$d_{i,j} = \left(\sum_p w_{i,p} (x_{i,p} - y_{i,p})^2 \right)^{\frac{1}{2}} \quad (1)$$

In the single choice model, the demand captured by product j is S_i if $d_{i,j} < d_{i,J}$ for all $j \neq J$. In the probabilistic choice model, the share of an ideal point's demand captured by a given product j is determined by the size of the choice set (k) and the relative distances of all available products. It is assumed that due to self interest, consumers are more likely to choose products closest to their ideal points (Aaker and Meyer, 1974).

The brand share for product j from the ith ideal point ($d_{i,j}$) is based on Eq. (2):

$$\pi_{i,j} = \begin{cases} \dfrac{(1/d_{i,j})}{\sum_k (1/d_{i,k})} & \text{for the } k \text{ closest products} \\ 0 & otherwise \end{cases} \quad (2)$$

To determine the demand for product j, the share from the ideal point ($\pi_{i,j}$) is multiplied by the sales potential of the ith ideal point (S_i).

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In an extensive simulation comparison, SMS showed that PRODSRCH performs better than the earlier algorithms of Albers and Brockhoff (1977) and Gavish et al. (1983) in situations where consumers allocate their demand probabilistically. In the single choice situation, one version of the Gavish et al. (1983) algorithm performs very well.

Another advantage of PRODSRCH is that it relies on a well tested general purpose non-linear programming algorithm known as QRMNEW (May, 1979). Consequently, the complexity of the problem is determined by the number of dimensions of the search space (product dimensions) rather than the number of ideal points and product positions. For MDS-based product positioning, PRODSRCH is currently considered to be best approach for the single product location problem (Green and Krieger, 1989, p. 132). Kannan (2006) proposed a Conjoint Analysis approach for solving the positioning of a product problem.

MODEL FORMULATION

Let $\pi_B$denote the number of existing brands in the market, $\pi_A$the number of determinant attributes (dimensions in the MDS joint space), and π_N the number of new products to be introduced.

$Y_j=\{y_{jp}\}$ is the modal perception of the *j*-th product on the *p*-th dimension;

$w_i=\{w_{ip}\}$ is the vector of dimension weights for the *i*-th segment;

$I_i=\{I_{ip}\}$ is the ideal point for the *i*-th market segment. It is assumed to be *finite*, but need not lie in the region where feasible products might be located;

d_{ij}= the weighted Euclidean distance from the *j*-th product to the *i*-th segment's ideal point;

s_i= the *i*-th segment's demand;

π_{ij}= the share of the *i*-th segment's demand allocated to the *j*-th product alternative.

$\pi_{ij}=f\left(d_{ij}^{-1}\right)$

and

$$\sum_{j=1}^{n_B} \pi_{ij} = 1$$

For each *i* = 1,2,…,n_M before entry,

and

$$\sum_{j=1}^{n_B+n_N} \pi_{ij} = 1$$

For each *i* = 1,2,…,n_M after entry.

The model posit various functions for the behavior of including probabilistic ones that depend upon distance from ideal point as well as the typical deterministic (first-choice) rule. They assume that the objective is to maximize incremental market share; they also allow for the possibility of cannibalization after introduction. In their notation,

Ψ_i = the set of *k* closest products before introduction;

x_i = the *i*-th firm's self-products before introduction;

π_{ij} = likelihoods of purchase before introduction of new products;

$x_n = \{x_{np}\}$ = the *n*-th new product's position (in the n_A attribute space R_{n_A}); *n* = 1,2,…, n_N ;

L = an arbitrarily large number; and

Ψ_i^* , x_i^* and π_{ij}^* are the after-entry (of n_N new products) equivalents of Ψ_i , x_i and π_{ij} respectively.

Then we formulate (and solve) the mixed integer nonlinear programming problem,

Maximize $\sum_{i=1}^{n_M}\left(\sum_{j\in x_i} u_i \pi_{ij}^* - \sum_{j\in x_i} \pi_{ij}\right) S_i$

Subject to

$$d_i^{(k)}(1-u_i) \le \left[\sum_{p=1}^{n_A} \left(I_{ij} - x_p\right)^2 w_{ij}\right]^{\frac{1}{2}} < d_i^{(k)} + L(1-u_1),$$

For all $x_n \in R_{n_A}$, and $i \in n_M$, where u_i is zero or one depending on whether (1) or not (0), a self product (existing or new, located at $\{x_p\}$), is among the *k* closest for the *i*-th segment.

MODELING A PLANNING PROBLEM FOR POSITIONING A NEW PRODUCT IN A MULTIATTRIBUTE SPACE

This is a marketing problem faced by a firm which wishes to position a new brand product in an existing product class. It is natural that an individual choice for his/her most preferred products are influenced essentially by the perceptions and values of the products (e. g. the design of the product). Individuals usually differ in their choice of an object out of an existing set, and they would also differ if asked to specify an ideal object. Due to these differences, the aim of the problem considered here is to optimally design a new product in order to attract the largest number of consumers

Mathematical Statement of the Problem. The mathematical programming formulation of the problem is due to Duran and Grossmann [5]. Let N be the number of consumers who are a representative sample of the common population for a certain price range of a product class. Else, let M be the number of an existing product (e. g. different brands of cars) in a market which are evaluated by consumers and are located in a multiattribute space of dimension *K*. We then define

z_{ik} - ideal point on attribute k for the ith consumer, $i = 1, \ldots, N; k = 1, \ldots, K$

w_{ik} - weight given to attribute k by the ith consumer, $i = 1, \ldots, N; k = 1, \ldots, K$

δ_{jk} - ideal point on attribute k for the ith consumer, $i = 1, \ldots, N; k = 1, \ldots, K$

Furthermore, a region (hyper ellipsoid) defining the distance of each consumers to the ideal point can be determined in terms of the existing product, in a way to produce a formulation such that each consumer will select a product which is closest to his/her ideal point. It was mentioned above that the objective of the problem is to optimally design a new product $(x_k, k = 1, \ldots, K)$ so as to attract the largest number of consumers.

Duran and Grossmann [5] have extended the scope of the positioning problem by introducing the revenue of the firm from the new product sales to consumer i (c_i) as well as a function f for representing the cost of reaching locations of the new product within an attribute space. Now, the objective of the problem would be to maximize the profits the firm. The binary variable (y_i) is introduced for each consumer to denote whether he/she is attracted by the new product or not.

Consider a positioning problem in which there are 10 existing products (M), 25 consumers (N) and attributes (K). The algebraic representation of such a problem can be written as follows.

$$\text{Maximize } F = \sum_{i=1}^{25} c_i y_i - 0.6x_1^2 + 0.9x_2 + 0.5x_3 - 0.1x_4^2 - x_5$$

Subject to

$$\sum_{k=1}^{5} w_{ik}(x_k - z_{ik})^2 - (1 - y_i)H \leq R_i^2, \qquad i = 1, \ldots, 25$$

$$x_1 - x_2 + x_3 + x_4 + x_5 \leq 10$$
$$0.6x_1 - 0.9x_2 - 0.5x_3 + 0.1x_4 + x_5 \leq 0.64$$
$$x_1 - x_2 + x_3 - x_4 + x_5 \geq 0.69$$
$$0.157x_1 + 0.05x_2 \leq 1.5$$
$$0.25x_2 + 1.05x_4 - 0.3x_5 \geq 4.5$$
$$2.0 \leq x_1 \leq 4.5$$
$$0.0 \leq x_2 \leq 8.0$$
$$3.0 \leq x_3 \leq 9.0$$
$$0.0 \leq x_4 \leq 5.0$$
$$4.0 \leq x_5 \leq 10.0$$
$$0 \leq y_i \leq 1 \text{ and integer } \forall_i$$

Where

$$R_i^2 = \min_{j=1,\ldots,10} \{\sum_{k=1}^{5} w_{ik}(\delta_{jk} - z_{ik})^2\}, \qquad i = 1, \ldots, 25$$

$$C^T = [1, 0.2, 1, 0.2, 0.9, 0.9, 0.1, 0.8, 1.0, 0.4, 1, 0.3, 0.1, 0.3, 0.5, 0.9, 0.8, 0.1, 0.9, 1, 1, 1, 0.2, 0.7, 0.7]$$

and $H = 1000$

The data for the coordinates of existing product (δ_{jk}), ideal points (z_{ik}) and attribute weights (w_{ik}) can be obtained in Duran and Grossmann (1986b).

It can be seen that the above formulation is a MINLP model and it contains 25 binary variables, 5 continuous bounded variables, 30 inequality constraints (25 of them acting nonlinearly) and a nonlinear objective function.

THE ALGORITHM

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \le f \le 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Calculate

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

this is a pricing operation

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T a_j$

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\sigma_{ij}} \right| \right\}$$

I. *For nonbasic j at lower bound*

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

II. *For nonbasic j at upper bound*

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{(1-\delta_{i^*})}{-\sigma_{ij}}$

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If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{(1-\delta_{i^*})}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4. Calculate

$\alpha_{j^*} = B^{-1}\alpha_{j^*}$

i.e. solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

If j^* lower bound

Let

$$A = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$$

$$B = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$$

$$C = \Delta$$

the maximum movement of j^* depends on: $\theta^* = \min(A, B, C)$

If j^* upper bound

Let

$$A' = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij^*}} \right\}$$

$$B' = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij^*}} \right\}$$

$$C' = \Delta$$

The maximum movement of j^* depends on: $\theta^* = \min(A', B', C')$

Step 6. Exchanging basis for the three possibilities

1. If A or A'
   - $x_{B_{i'}}$ becomes nonbasic at lower bound $l_{i'}$
   - x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
   - x_{i^*} stays basic (non-integer)
2. If B or B'
   - $x_{B_{i'}}$ becomes nonbasic at upper bound $u_{i'}$
   - x_{j^*} becomes basic (replaces $x_{B_{i'}}$)
   - x_{i^*} stays basic (non-integer)
3. If C or C'
   - x_{j^*} becomes basic (replaces x_{i^*})
   - x_{i^*} becomes superbasic at integer-valued

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Do integer lines search to improve the integer feasible solution

DISCUSSION

We solved the problem on PC with processor Intel(R) Core (TM) i5-2300 CPU @ 280 GHZ and RAM 4.00GB. We used our Nonlinear Programming software in order to get the optimal continuous solution. The results are presented in Table 1. It can be observed that five binary variables have had integer value (all of them are in upper bound). The binary variable y_i happens to be a superbasic in the continuous result with non-integer value. We moved this variable to its closest integer by using a truncation strategy and kept the integer result as superbasic. The corresponding basic variables would be affected due to this movement. Therefore it is necessary to check the feasibility of the results. The proposed integerizing algorithm was then implemented on the remaining non-integer binary variables. The integer results can also be found in Table 2.

It is interesting to note that our result $(F = 8.14313)$ is slightly better that Duran and Grossmann's [5] result $(F = 7.78913)$. The binary variable y_i has a value of 1.0 in our result instead of 0.0 as in Duran and Grossmann's result. The total computational time to get the integer result by using our proposed algorithm is 10.98 seconds.

Table 1. The Results of the Positioning Problem.

Variable	Activity in Cont.Soln.	Activity after integ. Process
x_1	2.0	2.0
x_2	8.0	7.81528
x_3	7.32849	6.29911
x_4	3.52381	3.56779
x_5	4.0	4.0
y_1	0.93153	1.0
y_2	0.70970	0.0
y_3	0.67548	0.0
y_4	0.50181	0.0
y_5	0.77537	0.0
y_6	1.0	1.0
y_7	0.78191	0.0
y_8	1.0	1.0
y_9	0.82922	0.0
y_{10}	0.11168	0.0
y_{11}	0.81785	0.0
y_{12}	0.74375	0.0
y_{13}	0.93852	0.0
y_{14}	0.61360	0.0
y_{15}	1.0	1.0
y_{16}	0.69117	0.0
y_{17}	1.0	1.0
y_{18}	0.91958	0.0
y_{19}	0.83079	0.0
y_{20}	0.97451	1.0
y_{21}	0.93383	0.0
y_{22}	0.57154	0.0

y_{23}	0.49858	0.0
y_{24}	0.91093	0.0
y_{25}	1.0	1.0
Obj.value(F)	16.41964	8.14313

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